State both parts of the Fundamental Theorem of Calculus, and the Net Change Theorem. **SCORE: ______ / 4 PTS**

IF f IS CONTINUOUS AND $g(x)=\int_a^x f(t)\,dt$ (WHERE a IS A CONSTANT), THEN $g'(x)=f(x)$

IF f IS CONTINUOUS ON $[a,b]$ AND F IS ANY ANTI DERIVATIVE OF f, THEN $\int_a^b f(x)\,dx = F(b)-F(a)$

IF F' IS CONTINUOUS ON $[a,b]$, THEN $\int_a^b F'(x)\,dx = F(b)-F(a)$

Answer the following questions about the definition of the definite integral as presented in lecture. **SCORE: ______ / 3 PTS**
(Your answers may refer to the fact that the definite integral equals the area under a curve which is above the $x-$axis.)

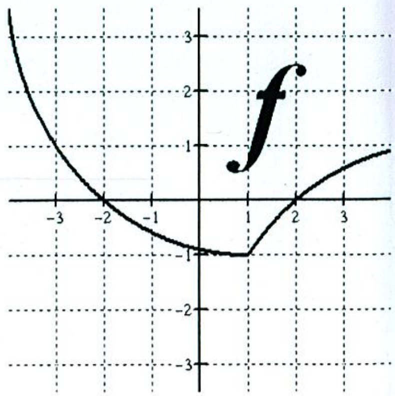
[a] What does the index of the limit (n) represent ? Why does the index of the limit approach the value that it does ?

n IS THE NUMBER OF SUBINTERVALS/RECTANGLES WE SPLIT THE ORIGINAL INTERVAL/AREA INTO. $n \to \infty$ TO GET A MORE + MORE ACCURATE APPROXIMATION OF THE AREA

[b] Why are the conditions at the end of the definition required ?
(In other words, if those conditions were not met, why would that present a problem for our definition ?)

THE LIMIT MUST EXIST SO $\int_a^b f(x)\,dx$ CAN HAVE A VALUE, AND IT MUST HAVE ONLY ONE VALUE, SO THE LIMIT BE THE SAME HOWEVER x_i^* ARE CHOSEN

Let $g(x) = \int_2^x f(t)\,dt$, where f is the function whose graph is shown on the right. **SCORE: ______ / 6 PTS**



[a] Find $g'(1)$. **Explain your answer very briefly.**

① $g'(1)=f(1)=-1$ ①

[b] Find all intervals over which g is concave up. **Explain your answer very briefly.**

① $g'=f$ IS INCREASING ON $[1,4]$ ①

[c] Find the $x-$coordinates of all local maxima of g. **Explain your answer very briefly.**

① $g'=f$ CHANGES FROM POSITIVE TO NEGATIVE AT $x=-2$

①

Water is flowing out of a garden hose into a swimming pool at a rate of $r(t)$ gallons per minute, where t is the number of minutes since 3 pm. What is the meaning of the equation $\int_{10}^{15} r(t)\,dt = 40$ in this situation ?

SCORE: ______ / 3 PTS

NOTES: **Your answer must use all three numbers from the equation, and along with appropriate units.**
Your answer should NOT use the words "integral", "antiderivative", "rate of change" or "derivative".

40 GALLONS OF WATER FLOWED OUT OF THE HOSE/INTO THE POOL FROM 3:10 PM TO 3:15 PM

If $k(x) = \int_{x^4}^{\cos x} \tan t^3\,dt$, find $k'(x)$.

SCORE: ______ / 5 PTS

$$k'(x) = \frac{d}{dx}\left[\int_0^{\cos x} \tan t^3\,dt + \int_{x^4}^{0} \tan t^3\,dt\right]$$

$$= \frac{d}{dx}\left[\int_0^{\cos x} \tan t^3\,dt - \int_0^{x^4} \tan t^3\,dt\right]$$

$$= \frac{d}{d\cos x}\int_0^{\cos x} \tan t^3\,dt \cdot \frac{d\cos x}{dx} - \frac{d}{dx^4}\int_0^{x^4} \tan t^3\,dt \cdot \frac{dx^4}{dx}$$

$$= \tan(\cos^3 x)\ 1\ (-\sin x)\ 1 - (\tan x^{12}) \cdot 4x^3$$

$$1 = -\sin x \tan(\cos^3 x) - 4x^3 \tan x^{12}\ 1$$

Evaluate the following integrals.

SCORE: ______ / 9 PTS

[a] $\int_1^2 \frac{(3-x)^2}{x^3}\,dx$

$$= \int_1^2 \frac{9-6x+x^2}{x^3}\,dx$$

$$1$$

$$= \int_1^2 (9x^{-3} - 6x^{-2} + x^{-1})\,dx$$

$$= \left(-\frac{9}{2}x^{-2} + 6x^{-1} + \ln|x|\right)\Big|_1^2$$

$$1 \quad 1 \quad 1$$

$$= -\frac{9}{2}\left(\frac{1}{4} - 1\right) + 6\left(\frac{1}{2} - 1\right) + \ln|2| - \ln|1|$$

$$= -\frac{9}{2} \cdot -\frac{3}{4} + 6\left(-\frac{1}{2}\right) + \ln 2$$

$$= \frac{27}{8} - 3 + \ln 2$$

$$= \frac{3}{8} + \ln 2 \quad 1\tfrac{1}{2}$$

[b] $\int \csc^2 x \cot^6 x\,dx$

$u = \cot x$

$\frac{du}{dx} = -\csc^2 x$

$-du = \csc^2 x\,dx$

$$1 = -\int u^6\,du$$

$$1 = -\frac{1}{7}u^7 + C$$

$$= -\frac{1}{7}\cot^7 x + C$$

$$1 \quad \tfrac{1}{2}$$
